Microwave Absorption by Conductor-Loaded Dielectrics

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***Abstract*—A theoretical model to predict the microwave absorption by a conductor-loaded dielectric is developed. The microwave power transmission coefficient of the test material is derived in terms of the effective conductivity and permittivity parameters (of the composite material) as a function of volume fraction of the conductor loading. The absorption/transmission characteristics of the test material versus volume fraction of metal loading are described by three distinct regimes having low-loss, lossy, and metal-like characteristics. Theoretical predictions are comparable with the measured data pertinent to an iron-plus-dielectric material.**

I. Introduction

Dielectric materials loaded with particulate, needle-like, or flaky conducting materials (such as graphite or nickel) find a variety of applications at microwave frequencies such as radar-absorbing materials, bioelectromagnetic phantoms, EMI shields, microwave absorbers for anechoic chambers, etc. The effective electromagnetic response of such mixture media with metallic inclusions is decided by: 1) volume fraction, geometrical shape, and conductivity of the metallic inclusions; and 2) complex permittivity of the host medium. The other deciding factors are the frequency of operation and stochastical aspects of random particulate dispersion in the mixture. (The volume fraction indicated refers to the volume content of a constituent material expressed as a fraction of the total volume of the mixture.)

In the static and/or quasistatic regimes, the earliest versions of conductor-loaded dielectric mixture formulations are due to Maxwell-Garnett [1] and Rayleigh [2]. These, however, apply to dilute concentration of the inclusions only. Subsequent studies include those due to Bruggeman [3], Lal and Parshad [4] Scarisbrick [5], Kusy [6], and Frame and Tedford [7]—all of which mostly refer to static or low-frequency applications, and/or apply to low-volume fractions or contain empirical parameter(s) to match the experimental data.

The electrical resistivity of binary composites at static conditions has also been modeled via a general effective media approach which combines the effective media theory (due to Bruggeman [3]) and percolation concepts—a review of which has been presented by McLachlan *et al.* [8]. In order to extend the conductivity model(s) of dielectric–conductor mixtures to time-varying excitations, the classical work of Maxwell-Garnett [1] has been used to depict the random metal–insulator composite as a polarizable medium in which the metal inclusions play the role of "atoms" [9]. Relevant studies as adopted by others as well [10]–[13] have yielded interesting results (experimental and/or theoretical) pertinent to the dielectric-loaded mixtures.

One of the authors developed elsewhere [13] a comprehensive analytical model based on complex susceptibility concepts to describe a conductor–dielectric mixture, and deduced its effective dielectric constant and conductivity parameters. Relevant formulations have taken into consideration the frequency dependence, the statistical nature of the mixture (via logarithmic law of mixing), and the influence of particulate geometry on dielectric polarization, and, above all, those analytical expressions are entirely nonempirical.

The present study is based on the above model [13], and deduces analytical description(s) of the power transmission coefficient of a dielectric–metal mixture at microwave frequencies. This transmission is characterized by three distinct regimes of insertion loss offered by the test sample, namely, dielectric-dominant, conductivity-dominant (metal-like), and an intermediate lossy range which depends heavily on the volume fraction of the inclusions. Relevant theoretical considerations and experimental studies are presented in the following sections.

II. Theoretical Considerations

As indicated earlier, expressions for the effective (relative) permittivity (ϵ_{eff}) and conductivity (σ_{eff}) of a dielectric–metal mixture at high frequencies have been derived in [13]. They are given by

$$\epsilon_{\text{eff}} = \{\epsilon_2/[1 + (\epsilon_2 - 1)^u]\} \cdot \{[(\sigma_1/\omega\epsilon_0)^\theta(\epsilon_2 - 1)^{1-\theta}\cos(\pi\theta/2)]^u + 1\} \tag{1a}$$

$$\sigma_{\text{eff}} = \sigma_1\{[\omega\epsilon_0(\epsilon_2 - 1)/\sigma_1]^{1-\theta}\sin(\pi\theta/2)\}^u \tag{1b}$$

where σ_1 is the conductivity of the inclusions (in siemens/meter), ϵ_2 is the relative permittivity of the host medium, ϵ_0 is the free-space permittivity equal to $(1/36\pi) \times 10^{-9}$ F/m, $\omega = 2\pi \times$ frequency, and θ is the volume fraction of the inclusions. Further, u depicts an order parameter of the mixture system, which for spherical particulate inclusions (with eccentricity $e = 0$) is deduced in [13] as 1/6 in terms of the Langevin function.

When $\theta \rightarrow 1$, the test material represents a pure conductor which would offer an attenuation constant (α_1) to electromagnetic propagation through it equal to $(1/\delta_1)$ where δ_1 is the skin depth of the metal. The specific power absorption (watts/meter^3) by the metal, namely, ($E^2\sigma_1$), is then proportional to $[1 - \exp(-2\alpha\delta_1)]$ where E is the incident electric field intensity (in volts/meter). Over the same depth (δ_1), electromagnetic power absorption in the mixture with a volume fraction (θ) of the metallic inclusions is ($E^2\sigma_{\text{eff}}$) proportional to $[1 - \exp(-2\alpha\delta_1)]$ where α refers to the attenuation coefficient of the mixture. Inasmuch as $\alpha_1\delta_1 = 1$ and the relative power loss in a unit volume of the mixture is the ratio $[(E^2\sigma_{\text{eff}})(\theta)/(E^2\sigma_1)(1)]$, α can be deduced as

$$\alpha = (\alpha_1/2)\ln\{[1 - x(\sigma_{\text{eff}}\theta/\alpha_1)]^{-1}\} \tag{2}$$

where $x = [1 - \exp(-2)] = 0.864665$ and σ_1 is explicitly given by $(\pi f \mu_1 \sigma_1)^{1/2}$ neper/meters, with μ_1 being the permeability of metal inclusions.

Considering a slab of thickness d (meters) of the mixture medium with a volume fraction (θ) of the conducting inclusions, a power transmission attenuation function $F(\theta, u)$ can be defined and deduced as follows.

Let T be the power transmission coefficient specified by $T = T_1T_2T_3$ where T_1 refers to the transmission factor for the normal incidence of a plane wave at a medium of relative permittivity ϵ_{eff}. That is, $T_1 = [2\sqrt{\epsilon_{\text{eff}}}/(1 + \sqrt{\epsilon_{\text{eff}}})]^2$. The factor T_2 accounts for the attenuation over the thickness (d) of the medium, and is given by $[1 - \exp(-\alpha d)]^2$. Inasmuch as the medium on which the wave is incident is of finite thickness (d), the overall transmission coefficient could be influenced by multiple reflections at the air–composite interfaces. Nicolson and Ross [14] have deduced the reflection/transmission coefficient(s) with the inclusion of such multiple reflections. A similar

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approach as presented in [15] provides a reflection correct factor T_3 equal to $|(K-1)/(K+1)\exp(-\gamma d)|^2$ where K is the ratio of wave impedances corresponding to free space and the test material. Explicitly, it is given by $K = [120\pi/(1+j)R_s]$. Here, R_s is the surface resistance of the test material equal to $(\alpha/\sigma_{\text{eff}})\ \Omega$; further, $\gamma = \alpha + j\beta$, with $|\alpha| = |\beta|$.

A transmission attenuation function can now be defined in terms of T as

$$F(\theta, u) = (1 - TX) \tag{3}$$

where the factor $X = \tau_1/\tau_\theta$ and τ_1 and τ_θ are the rearrangement times of the mixture material at $\theta \to 1$ and at θ, respectively. The rearrangement time τ refers to the permittivity/conductivity ratio of the material, and depicts the time involved in the rearrangement of charges placed in the medium in appearing as surface charges. For a perfect conductor, $\tau \to 0$, and in a perfect dielectric (insulator), $\tau \to \infty$. The rearrangement time is a convenient way to differentiate between insulators and conductors. Accordingly, the mixture medium can be classified into three regimes as follows: 1) low-loss region I with τ_θ being large, 2) intermediate-lossy region II, and 3) high-loss region III with τ_θ being small. Pertinent to these three regions, the corresponding transitional volume fractions can be evaluated explicitly as detailed later.

Considering region I, microwave propagation through the material sample can be expected to have a power transmission coefficient Γ_T (normalized with respect to power transmission coefficient, namely, $[(2\sqrt{\epsilon_2})/(1+\sqrt{\epsilon_2})]^2$ specified by the pure dielectric host medium) being close to unity as dictated by the low-loss characteristics of the medium. This implies that the loss is due to reflection at the air–composite interface only. As θ increases, Γ_T should decrease as governed by (3). Likewise, for region III, Γ_T should tend towards zero with an increase in θ as decided by (3). Hence, the following formulations are derived:

$$\begin{aligned}\Gamma_T &= (3/4)F(\theta, u=1/3) + (1/4)F(\theta, u=1/6)\\ &= 0.75 + 0.25F(\theta, u=1/6) \qquad \text{for region I}\end{aligned} \tag{4a}$$

$$\begin{aligned}\Gamma_T &= (3/4)[1 - F(\theta, u=1/3)] + 1/4F(\theta, u=1/6)\\ &= 0.25F(\theta, u=1/6) \qquad \text{for region III}\end{aligned} \tag{4b}$$

where $F(\theta, u=1/3) \to 1$ for all values of θ.

The intermediate section (region II) covers a narrow range of volume fraction over which a low-loss to high-loss abrupt transition prevails. Such a switching behavior is controlled by the stochastical nature of particulate dispersion in the test material; and the complex dielectric constant of a metal–insulator mixture exhibits a divergent behavior or a singularity in the vicinity of a percolation threshold value (θ_c) of the metallic volume fraction [9]–[16].

In view of the above discussion, a deterministic law of variation for region II predicting the transmission coefficient versus the volume fraction cannot be per se stipulated. However, such a region can be bounded as constrained by certain limiting values of θ, and, for any given sample, the transitional values of Γ_T versus θ will be confined to this bounded area. Such specification of conductivity and/or permittivity of binary mixtures within certain stochastical bounds is not uncommon [12].

Referring to Fig. 1, this bounded region (region II) is specified by the corner coordinates $A(0, 1)$, $B(2u/2, 1)$, $C(2u/2, 0)$, and $D(4u/2, 0)$. The transitional volume fractions corresponding to A, B, C, and D indicate the critical behavior vis-à-vis the lossy nature of the test material. Such a behavior is also specified in terms of the order parameter (u) portraying a transitional characteristic analogous to dipole polarization (parallel or antiparallel) transitions. The limit-switching between low-loss to high-loss characteristics as controlled by θ over the limits A, B, C, and D can therefore be decided by the Langevin–Debye theory of dipole polarization. Depicting D as the upper (saturation) limit corresponding to $(1-\Gamma_T) \to 1$, then it is determined by an order parameter $4u/2$. The lower (antisaturation) limit corresponds to A with $(1-\Gamma_T) \to 0$ and has a value $0(u/2)$. Further, $\theta_{\max}$ in Fig. 1 is the extreme limit of θ beyond which the mixture is metal-like. At this value of $\theta_{\max}$, $\Gamma_T \to 0$. The arithmetic mean of $\theta = 0$ and $\theta = 1$ statistically predicts $\theta_{\max}$ as equal to the mean value of these extreme (0 and 1), namely, 0.5. Further, from the considerations of Langevin dipole theory [13], the order parameter u for spherical particulates is taken as 1/6.

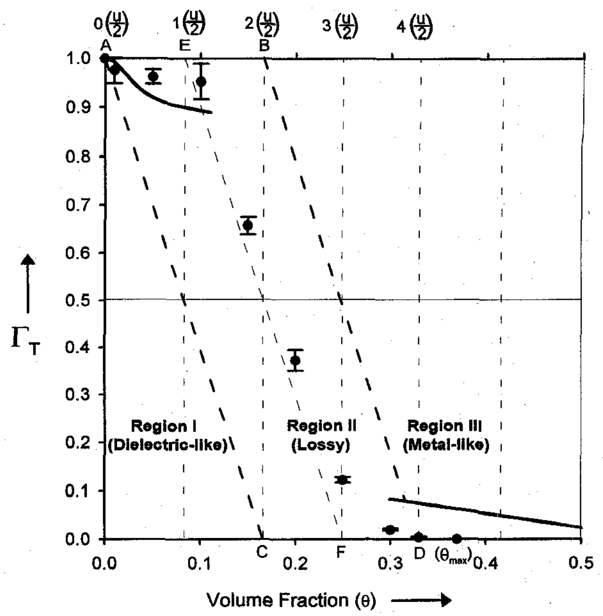

Fig. 1. Power transmission coefficient (Γ_T) versus volume fraction (θ) of conducting inclusions. Constituents of the test samples: host–medium ($CaCO_3$): $\epsilon_2 = 5.0$, conducting inclusions (Fe): $\sigma_1 = 10$ S/m, shape of the inclusions: spherical ($u = 1/6$), • • • • measured data at 9.6 GHz. (Γ_T: normalized with respect to power transmission coefficient corresponding to the sample being a totally dielectric (host) medium.)

III. Experimental Study

Microwave power transmission coefficient (Γ_T) versus volume fraction (θ) of conducting inclusions was measured for a test material constituted of commercial grade, finely divided iron particles of 6–9 μm sizes ($\ll \lambda_0$, λ_0 = free-space wavelength) dispersed in a dielectric ($CaCO_3$) host material. Several samples were made by changing the volume fractions of iron from 0.01– 0.6. In each case, the sample was a pellet (compressed through 6 tons with a pelletizing compressor) of diameter 28.6 mm and thickness (d) 4.0 mm. The sample was placed in a corresponding circular window cut out of a metal screen of about 20 × 20 cm. A simple transmitting and receiving horn arrangement with relevant microwave plumbing (Fig. 2) was used to measure the insertion loss offered by each sample (mounted on the metal screen) and interposed between the horns. The transmitter horn aperture size was 2.8 × 2.8 cm and that of the receiving horn was 6 × 5 cm. Denoting the horn aperture dimensions as $D \times D$, the sample was located in the interspace between the horns at a spacing from each of the horn aperture(s) above $\pi D^2/\lambda_0$ (where λ_0 is the free-space wavelength). This spacing approximately enables a plane-wave excitation of the sample. The power transmission coefficient was ascertained at an X-band frequency (9.6 GHz) by measuring the relative power received

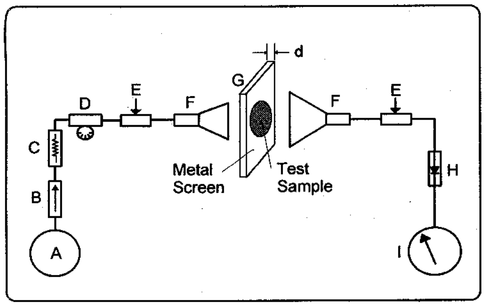

Fig. 2. Measurement of power transmission coefficient of the test samples at X-band frequencies. A: X-band microwave source. B: Isolator. C: Attenuator. D: Frequency meter. E: Slide-screw tuner. F: Horn antenna. G: Test composite. H: Diode detector. I: SWR meter.

with the apertured screen interposed and with the aperture on the screen filled with the test sample. The power levels were set such that the detector at the receiving end operates in the square-law regime. The detected signal was measured with an HP415E SWR meter on an expanded decibel scale, as well as being monitored for comparison on a voltage scale at the output of a tuned linear amplifier. (The transmitted microwave signal was pulse-modulated at 1000 Hz). Measurements repeated on each sample four–five times yielded consistent results with a deviation of less than $\pm 5\%$. With the plane-wave excitation, it could be anticipated that the finite-sized aperture may lead to edge diffraction and introduce corresponding errors in the transmission measurements carried out. Since the measurements involved were relative, cancellation of such errors were presumed. The measurements with the pyramidal horns as described earlier were repeated by replacing the transmitter horn with a focused microwave Gaussian-beam launching arrangement as described in [17] by one of the authors. The corresponding measured results on the insertion loss differed from those pertinent to plane wave excitation only to a maximum extent of $\pm 3\%$. The spot size of the beam at the sample was approximately 2.5 cm when the sample was placed at 6 cm in front of the spherical lens during the measurements.

IV. Results and Discussion

Depicted in Fig. 1 are the average of measured data (with the spread about the mean value indicated by the error bars) on Γ_T versus θ. For the material constants of iron and $CaCO_3$ as indicated in Fig. 1, the calculated results on Γ_T via (4a) and (4b) are also shown in Fig. 1, along with the transition regions.

Considering the results presented, it can be observed that the theoretical predictions are comparable to the experimental results confirming the algorithmic approach pursued for regions I and III. For region II, the experimental data are within the stochastic bounds of transition with the limits posed by A, B, C, and D.

The present formulations, with the limited approximations, have no empirical parameters. Material constants, order parameter(s), volume fraction, and frequency solely decide the algorithmic representation(s) of Γ_T.

The results of the present study refer only to spherical particles dispersed with isotropic randomness. If the particulates have an eccentricity $e \neq 0$ and/or the particulate dispersion has an orderly texture, the order parameter $u = 1/2[L(e)/e]$ will change accordingly [13]. Here, $L(x)$ represents the Langevin function as detailed in [13]. The values of Γ_T versus θ in the transitional regime for nonspherical particles and/or anisotropic dispersions will still be confined to the bounding limits of $ABCD$, except that such values will approach towards the edges of $ABCD$, namely, AB or BD in Fig. 1. The choice of the edge will depend on the extreme parallel or antiparallel anisotropicity of the particulate arrangement and/or the fiber-like or disk-like particulate shape as dictated by the limiting values of particle-shape eccentricity. Relevant theoretical and experimental studies are in progress.

References

[1] J. C. Maxwell-Garnett, "Colours in metal glasses and metal films," *Phil. Trans. A. Roy. Soc. (London)*, vol. 203, pp. 385–420, 1904.

[2] L. Rayleigh, "On the influence of obstacles arranged in rectangular order upon the properties of a medium," *Phil. Mag.*, vol. 34, pp. 481–502, 1892.

[3] D. A. G. Bruggeman, "Berechnung verschiedenier physikalischen konstanten von heterogenen substanzen," *Ann. Physik.*, vol. 24, pp. 636–679, 1935.

[4] K. Lal and R. Parshad, "Permittivity of conductor-dielectric heterogeneous mixtures," *J. Phys. D: Appl. Phys.*, vol. 6, pp. 1788–1792, 1973.

[5] R. M. Scarisbrick, "Electrically conducting mixtures," *J. Phys. D: Appl. Phys.*, vol. 6, pp. 2095–2110, 1973.

[6] A. Kusy, "Chains of conducting particles that determine the resistivity of thick resistive films," *Thin Solid Films*, vol. 43, pp. 243–250, 1977.

[7] R. I. Frame and D. J. Tedford, "Long-term electrical conduction in films of alkyd-resin in graphite mixture," *IEEE Trans. Elec. Insul.*, vol. EI-21, no. 1, pp. 23–29, 1986.

[8] D. S. McLachlan, M. Blaszkiewicz, and R. E. Newnham, "Electrical resistivity of composites," *J. Amer. Ceram. Soc.*, vol. 73, no. 8, pp. 2187–2203, 1990.

[9] P. Sheng, "Microstructure and dielectric properties of granular composite films," *Opt. Laser Technol.*, vol. 13, no. 5, pp. 253–260, 1981.

[10] R. W. Cohen, G. D. Cody, M. D. Coutts, and B. Abeles, "Optical properties of granular silver and gold films," *Phys. Rev. B*, vol. 8, no. 8, pp. 3689–3701, 1973.

[11] D. S. McLachlan, A. Priou, I. Chenerie, E. Issac, and F. Henry, "Modeling the permittivity of composite materials with a general effective medium equation," *J. Electromagn. Waves Appl.*, vol. 6, no. 9, pp. 1099–1131, 1992.

[12] W. T. Doyle and I. S. Jacobs, "Effective cluster model of dielectric enhancement in metal-insulator composites," *Phys. Rev. B*, vol. 42, no. 15, pp. 9319–9327, 1990.

[13] P. S. Neelakanta, "Complex permittivity of a conductor-loaded dielectric," *J. Phys.: Condens. Matter*, vol. 2, pp. 4935–4947, 1990.

[14] A. M. Nicolson and G. F. Ross, "Measurement of the intrinsic properties of materials by time-domain techniques," *IEEE Trans. Instrum. Meas.*, vol. IM-19, no. 4, pp. 377–382, 1970.

[15] D. R. J. White, *A Handbook on Electromagnetic Shielding Materials and Performance.* Gainesville, VA: Don White Consultants, Inc., 1980, pp. 1–29.

[16] D. M. Grannan, J. C. Garland, and D. B. Tanner, "Critical behavior of the dielectric constant of a random composite near the percolation threshold," *Phys. Rev. Lett.*, vol. 46, no. 5, pp. 375–378, 1981.

[17] P. S. Neelakantaswamy, K. K. Gupta, and D. K. Banerjee, "A Gaussian-beam launcher for microwave exposure studies," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 426–428, 1977.